

MESTRADO INTEGRADO EM ENGENHARIA
ELETROTÉCNICA E DE COMPUTADORES

FACULDADE DE ENGENHARIA DA UNIVERSIDADE DO PORTO

Análise de Circuitos

Caderno de Exercícios

Eletrónica I

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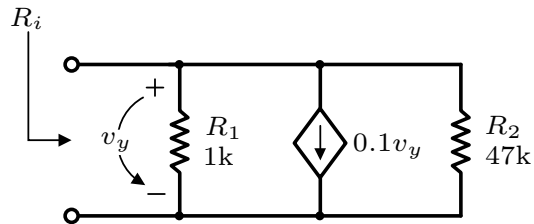
Versão 1.4

Março, 2016

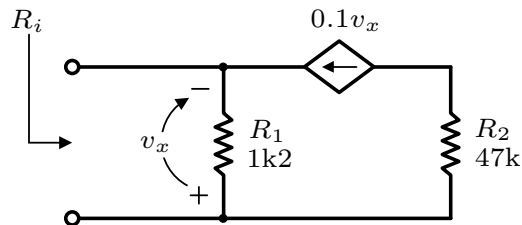
1. Enunciados dos Exercícios Resolvidos

1.1 Nos seguintes circuitos, recorrendo ao teorema da absorção da fonte, determine a resistência R_i conforme assinalada nas respetivas figuras.

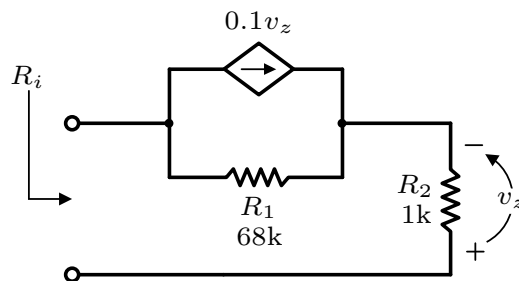
a



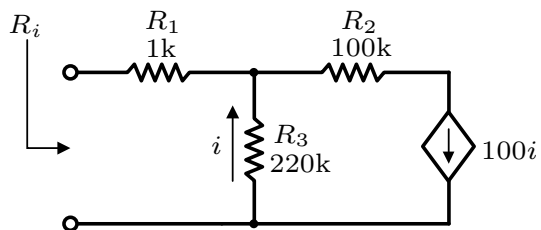
b



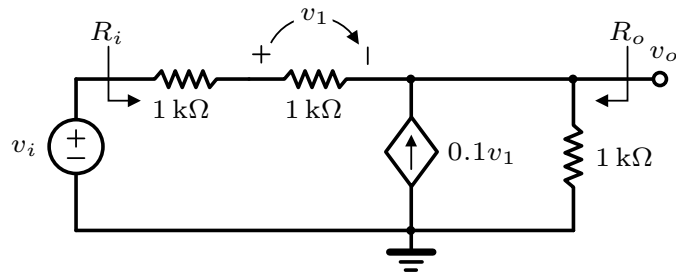
c



d



1.2 Considere o circuito da figura.

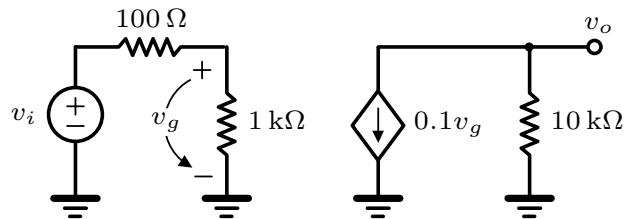


- a) Determine o ganho em tensão, $A_v = v_o/v_i$.
- b) Determine a resistência de entrada (R_i) vista pela fonte, bem como a resistência de saída (R_o), conforme indicadas na figura.

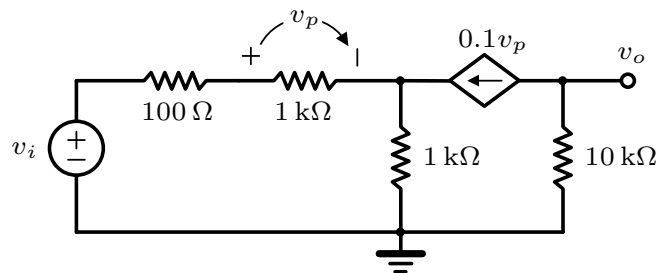
MT1, 20 MAR 2013, Q1.

1.3 Determine o ganho $A_v = v_o/v_i$ para cada uma das seguintes configurações.

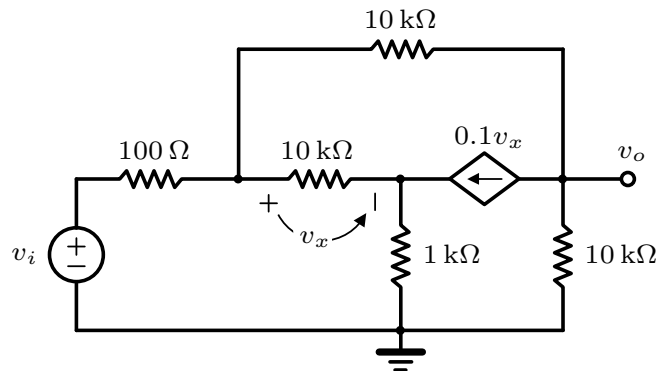
a)



b)

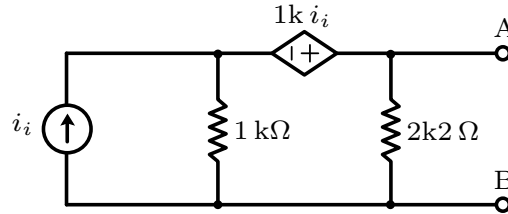


c)

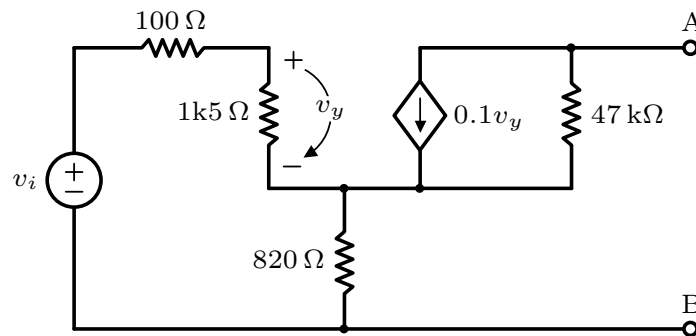


1.4 Para cada um dos esquemas representados a seguir, determine o circuito equivalente de Thévenin entre os nós A e B.

a)

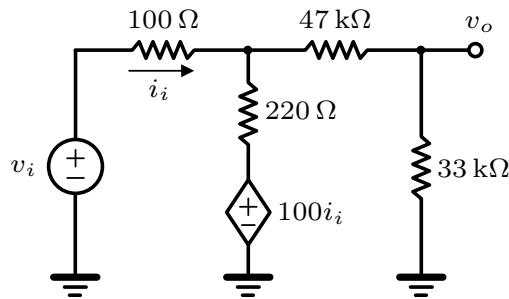


b)

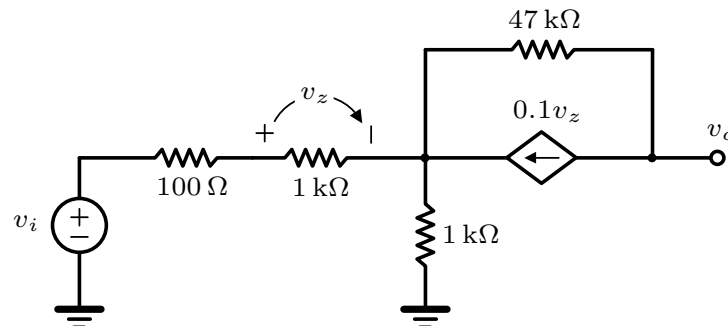


1.5 Determine os circuitos equivalentes de Norton dos esquemas seguintes, vistos a partir do nó v_o .

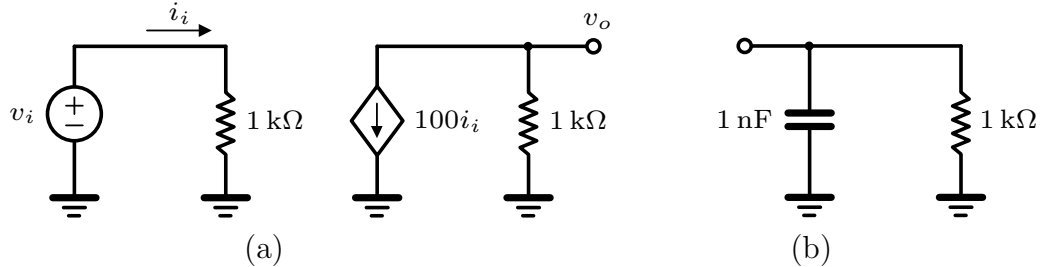
a)



b)



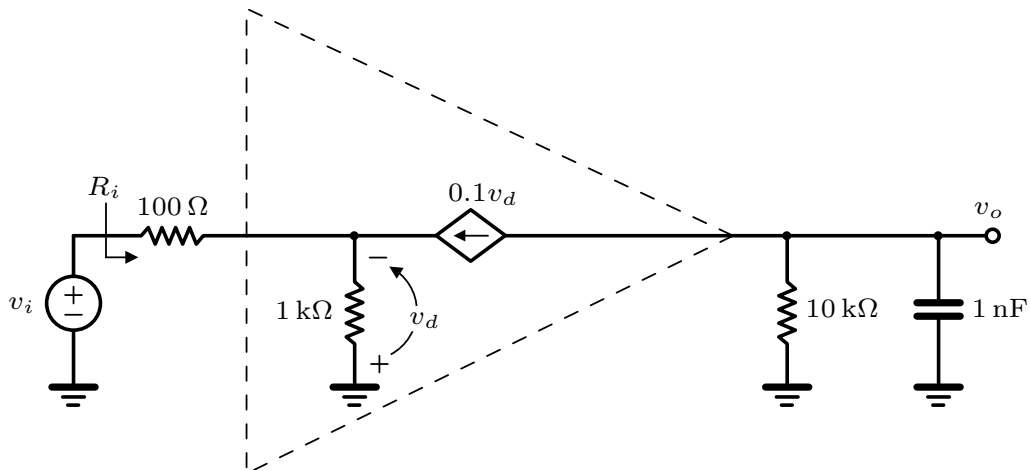
- 1.6 Considere o esquema representado na figura (a) que corresponde ao modelo para sinal de um amplificador.



- a) Calcule o ganho em tensão v_o/v_i e desenhe o equivalente de Thévenin do circuito, visto da sua saída.
- b) Suponha agora que é ligada a carga indicada na figura (b) e que v_i é um sinal em degrau de 0 para 100 mV. Desenhe a forma de onda na saída, cotando devidamente as amplitudes e tempos de resposta do circuito. **Nota:** Tenha em atenção que o produto $[frequência\ limite\ da\ banda] \times [tempo\ de\ subida] \simeq 0.35$.

MT1, 7 ABR 2010, Q1

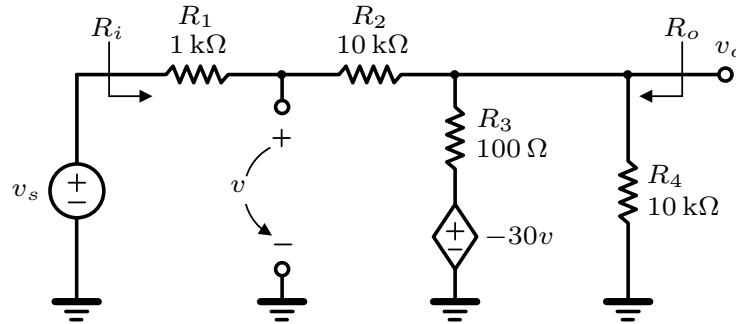
- 1.7 Considere o circuito da figura, em que dentro do triângulo se encontra o modelo equivalente, para sinal, de um amplificador.



- a) Determine a resistência de entrada (R_i) vista pela fonte de sinal v_i .
- b) Calcule o ganho v_o/v_i em função de $j\omega$ e desenhe o respetivo diagrama de Bode, de amplitude (devidamente cotado).

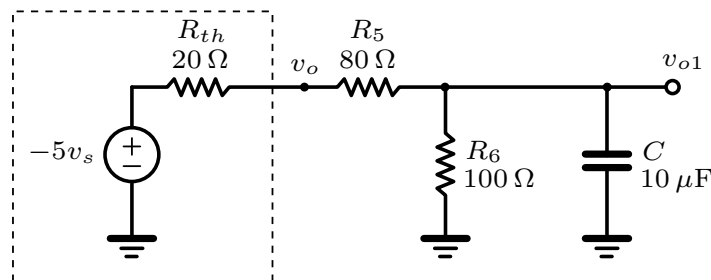
MT1, 2 ABR 2008, Q1

- 1.8 O circuito da figura seguinte representa um amplificador de tensão. Na resolução das alíneas seguintes faça as aproximações que julgar adequadas, mas justifique.

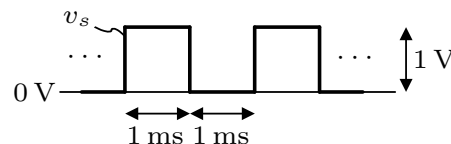


- a) Calcule a resistência de entrada R_i e de saída R_o do respetivo amplificador de tensão, conforme indicado na figura acima.
- b) Determine o ganho em tensão do amplificador $A_v = v_o/v_s$.

Independente do resultado obtido nas alíneas anteriores, assuma que o equivalente de Thévenin do amplificador de tensão é aquele apresentado na figura seguinte, a tracejado, ao qual se ligou um circuito RC.



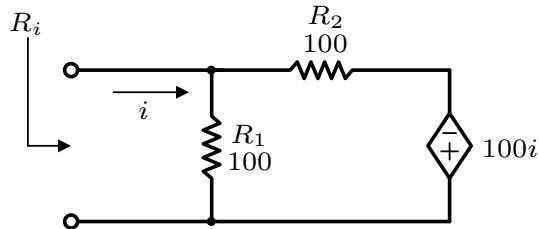
- c) Determine a função de transferência $H(j\omega) = V_{o1}(j\omega)/V_s(j\omega)$ e desenhe os respetivos diagramas assintóticos de Bode de amplitude e fase.
- d) Determine o valor da constante de tempo e esboce a forma de onda na saída v_{o1} , em regime permanente, se o sinal de entrada, v_s , for a onda quadrada periódica representada na figura seguinte. Deve cotar devidamente a forma de onda, indicando os valores de tensão máximo e mínimo.



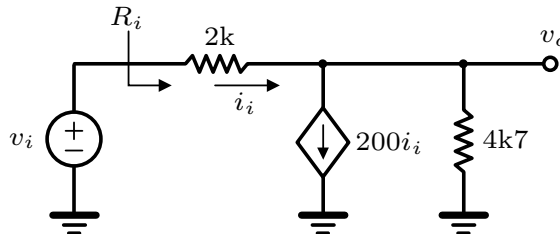
Nota: Recorde que a tensão aos terminais do condensador, na resposta a um degrau, segue $v_C(t) = V_{\text{final}} - (V_{\text{final}} - V_{\text{inicial}}) \cdot e^{-t/\tau}$.

2. Exercícios Propostos

2.1 Determine R_i no seguinte circuito.

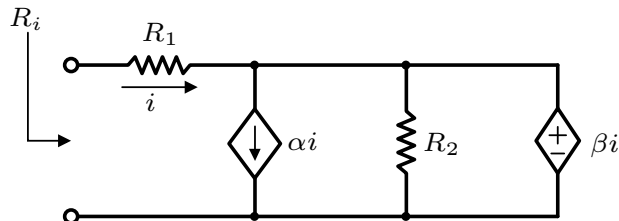


2.2 O circuito da figura representa o esquema equivalente de um amplificador. Determine a resistência de entrada $R_i = v_i/i_i$.

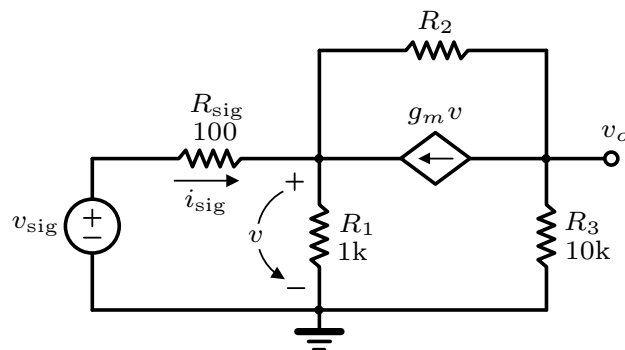


MT1, 18 OUT 2006, Q2

2.3 Recorrendo ao princípio da absorção da fonte, mostre que $R_i = R_1 + \beta$.



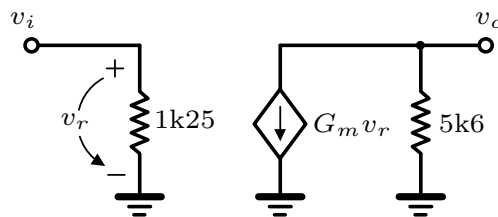
2.4 Considere o circuito da figura, em que as resistências têm o valor indicado e $g_m = 100 \text{ mA/V}$.



- a) Comece por considerar que R_2 não existe (i.e. $R_2 \rightarrow \infty$). Determine o ganho de tensão, $A_v = v_o/v_{sig}$.
- b) Considere agora que $R_2 = 20\text{ k}\Omega$ e calcule o valor da resistência de entrada vista pela fonte de sinal.

MT1, 25 MAR 2015, Q1

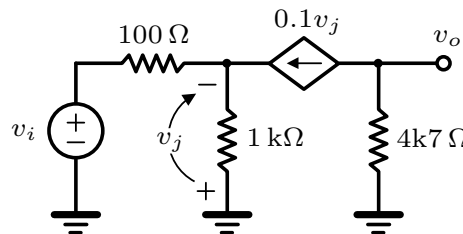
- 2.5] O circuito da figura representa o esquema equivalente de um amplificador, visto como amplificador de transcondutância (i.e., cuja fonte controlada é uma fonte de corrente). Calcule e desenhe o esquema equivalente do mesmo amplificador, visto como amplificador de tensão (i.e., cuja fonte controlada é uma fonte de tensão). Em particular, indique o valor do ganho de tensão em circuito aberto, A_{v_o} , sabendo que a transcondutância em curto-circuito é $G_m = 80\text{ mA/V}$.



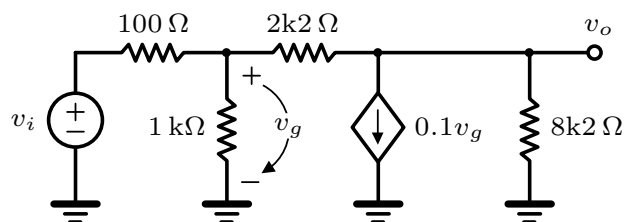
MT1, 18 OUT 2006, Q1

- 2.6] Determine o ganho $A_v = v_o/v_i$ para os circuitos das figuras seguintes. Use para o efeito as aproximações que achar conveniente (de forma justificada).

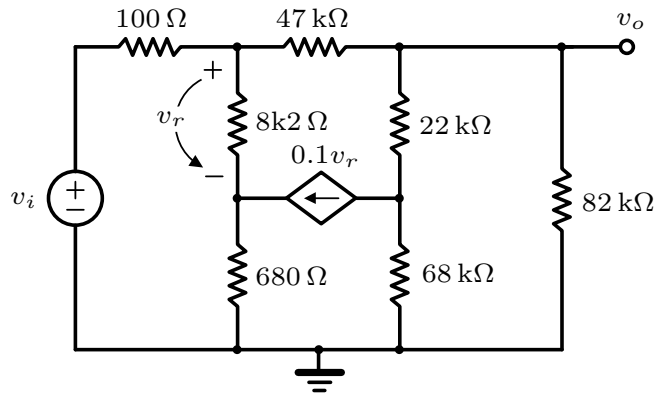
a)



b)

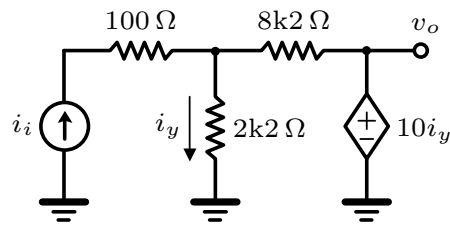


c

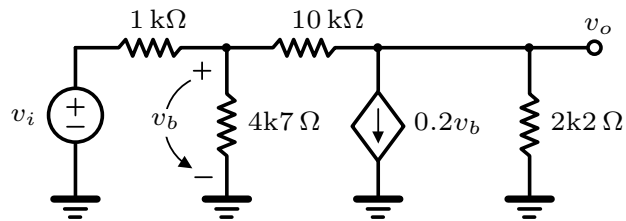


2.7 Reduza os seguintes circuitos aos equivalentes de Norton e/ou de Thévenin (conforme prefira), visto do nó v_o .

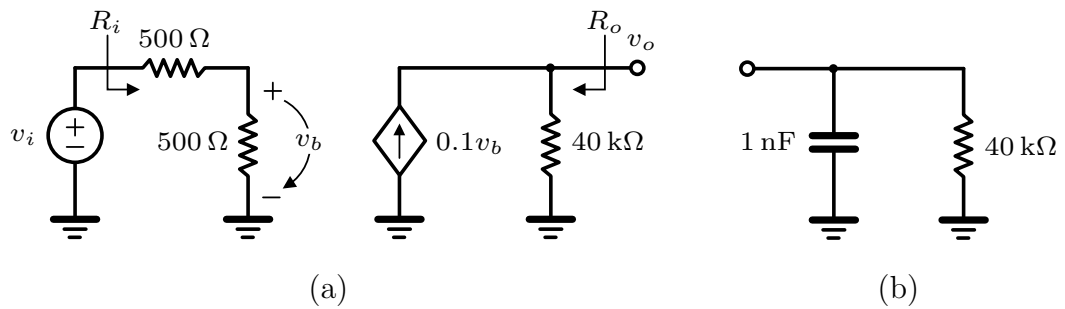
a



b



2.8 Considere o esquema representado na figura (a), que corresponde ao modelo para sinal de um amplificador.

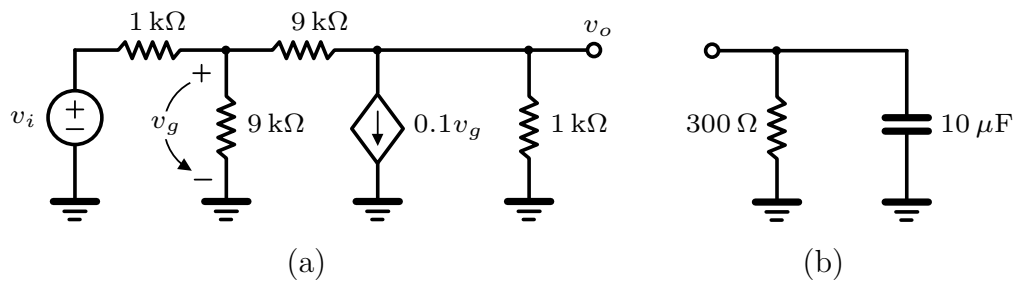


a Calcule o ganho de tensão $A_v = v_o/v_i$, bem como as resistências de entrada e de saída, respectivamente, R_i e R_o .

- b] Suponha agora que é ligada a carga indicada na figura (b). Determine o novo valor do ganho de tensão, $A_v(s) = V_o(s)/V_i(s)$ e desenhe os correspondentes diagramas de Bode de amplitude e fase, ambos devidamente cotados.

MT1, 23 OUT 2003, Q1

- 2.9] Considere, inicialmente, apenas o circuito da figura (a).



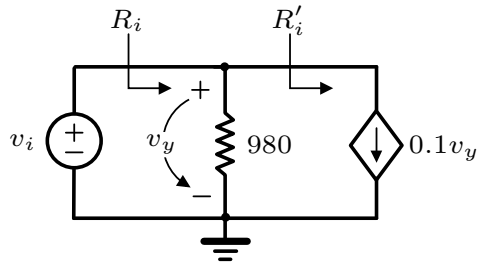
- a] Determine a resistência R_{th} do equivalente de Thévenin desse circuito, visto da saída.
- b] Suponha agora que liga o circuito da figura (b) na saída do circuito anterior. Desenhe de modo aproximado, a forma de onda obtida na saída (devidamente cotada em amplitude e tempo, mas ignorando o nível contínuo) como resposta a um degrau de 0 V para 1 V na entrada, recordando que o tempo de subida está relacionado com a constante de tempo do circuito por $t_s = 2.2\tau$, em que τ é a constante de tempo do circuito. Suponha que $V_{th} = -10v_i$ e, se não respondeu à alínea anterior, admita que $R_{th} = 100 \Omega$.

MT1, 21 MAR 2012, Q1

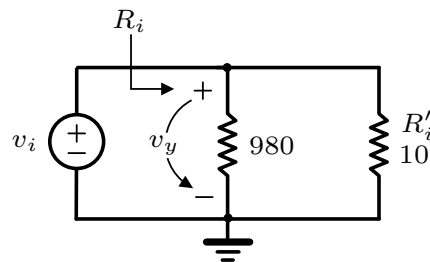
3. Resoluções

1.1

- a) • $R_1 \parallel R_2 = 1k \parallel 47k \simeq 980 \Omega$;

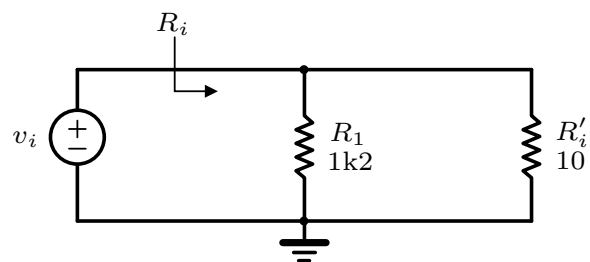
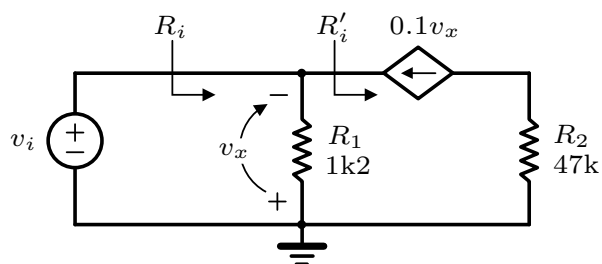


- $R'_i = v_y / (0.1v_y) = 10 \Omega$;



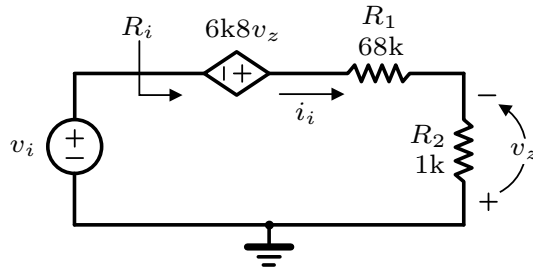
$\therefore R_i = 980 \parallel R'_i \simeq 10 \Omega$.

- b) • $R'_i = \frac{-v_x}{-0.1v_x} = 10 \Omega$;

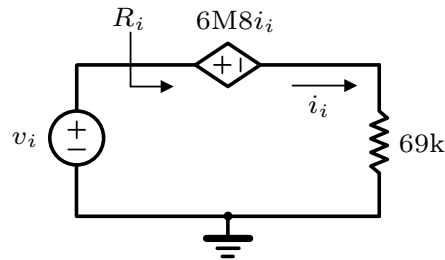


$\therefore R_i = R_1 \parallel R'_i = 1k2 \parallel 10 \simeq 10 \Omega$.

- c) • Norton \rightarrow Thévenin;

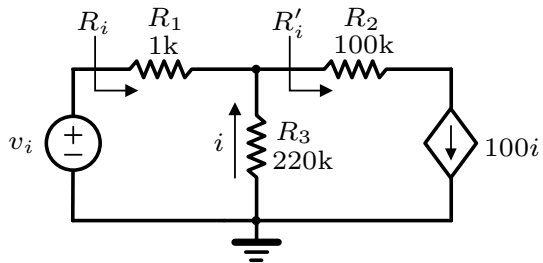


• $v_z = -R_2 i_i = -1k i_i$;

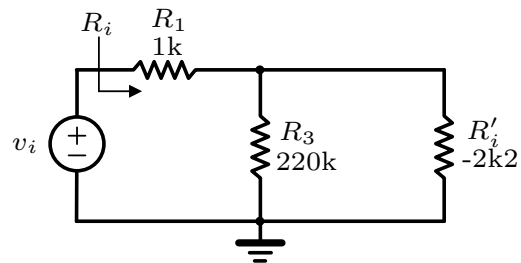


$\therefore R_i = 6M8 + 69k \simeq 6M87$.

- d) • $R'_i = -220ki/(100i) = -2k2i$;



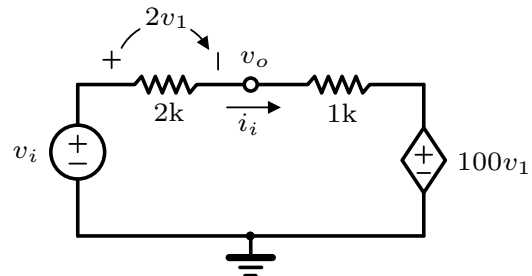
• $R'_i \parallel R_3 = -2k2 \times 220k / (220k - 2k2) \simeq -2k2$;



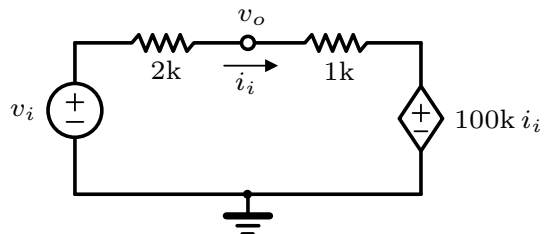
$\therefore R_i = 1k - 2k2 \simeq -1k2$.

1.2

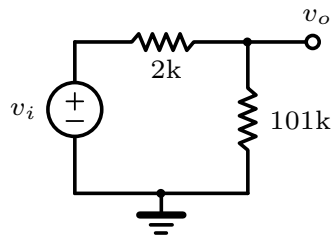
- a) • Norton \rightarrow Thévenin;



- $v_1 = 1\text{k} i_i$;



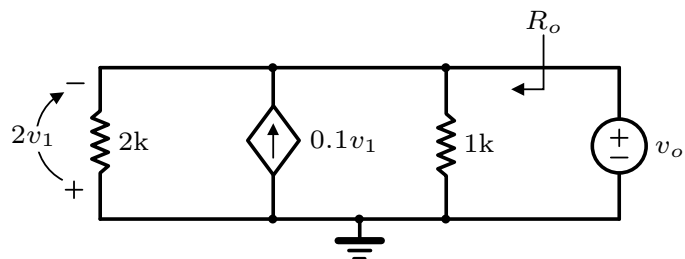
- Absorção da fonte dependente de tensão;



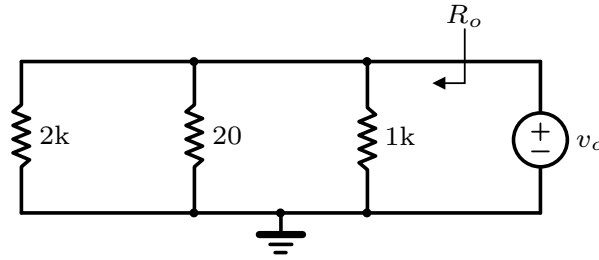
$$\therefore A_v = v_o/v_i = 101\text{k}/103\text{k} \simeq 0.98 \text{ V/V.}$$

- b) • Pela alínea anterior, $R_i = 103 \text{ k}\Omega$.

- Anula-se a entrada v_i e simplifica-se o circuito;



- Teorema da absorção da fonte: $R = -2v_1/(-0.1v_1) = 20 \Omega$;

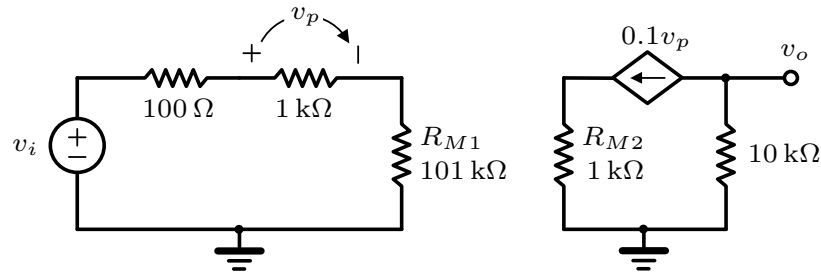


$$\therefore R_o = 2k \parallel 20 \parallel 1k = 1/(1/2k + 1/20 + 1/1k) \simeq 19.4 \Omega.$$

1.3

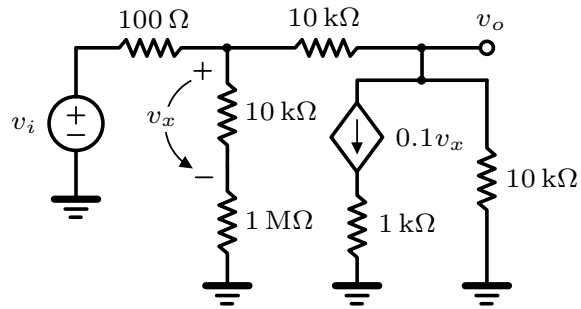
- a
- $v_o/v_g = -0.1 \cdot 10k = -1000 \text{ V/V}$;
 - $v_g/v_i = 1k/(100 + 1k) \simeq 0.9 \text{ V/V}$;
- $$\therefore A_v = v_o/v_i = (v_o/v_g) \cdot (v_g/v_i) = -900 \text{ V/V}.$$

- b
- Pelo dual do teorema de Miller:
 - $\alpha = 0.1v_p/(v_p/1k) = 100 \text{ A/A}$;
 - $R_{M1} = 1k(1 + \alpha) = 101 \text{ k}\Omega$;
 - $R_{M2} = R_{M1}/\alpha \simeq 1 \text{ k}\Omega$.

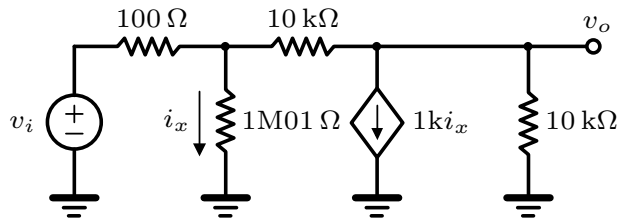


- $v_p/v_i = 1k/(100 + 1k + 101k) \simeq 9.89 \text{ mV/V}$;
 - $v_o/v_p = -0.1 \cdot 10k = -1000 \text{ V/V}$;
- $$\therefore A_v = v_o/v_i = (v_p/v_i) \cdot (v_o/v_p) = -9.89 \text{ V/V}.$$

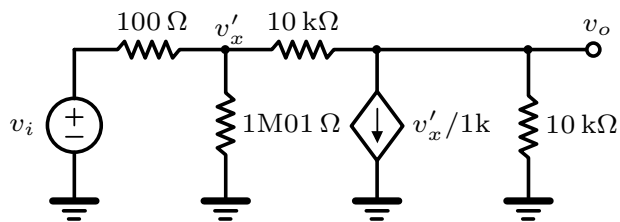
- c
- Pelo dual do teorema de Miller:
 - $\alpha = 0.1v_x/(v_x/10k) = 1000 \text{ A/A}$;
 - $R_{m1} = 1k \cdot (1 + \alpha) \simeq 1 \text{ M}\Omega$;
 - $R_{m2} = R_{m1}/\alpha = 1 \text{ k}\Omega$.



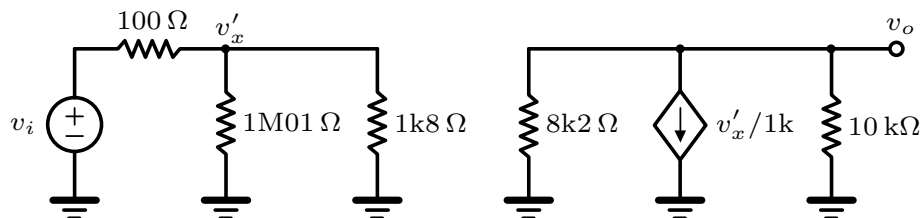
- Mudança de variável de controlo v_x :
 - $v_x = 10k \cdot i_x$;
 - $0.1v_x = 1k \cdot i_x$.



- Mudança de variável de controlo i_x : $i_x = v'_x/1M01 \rightarrow 1ki_x \simeq v'_x/1k$;



- Pelo teorema de Miller:
 - $v_o/10k = (v'_x - v_o)/10k - v'_x/1k \Leftrightarrow \gamma = v_o/v'_x = -4.5 \text{ V/V}$;
 - $R_{M1} = 10k/(1 - \gamma) \simeq 1k8 \Omega$;
 - $R_{M2} = 10k\gamma/(\gamma - 1) \simeq 8k2 \Omega$.

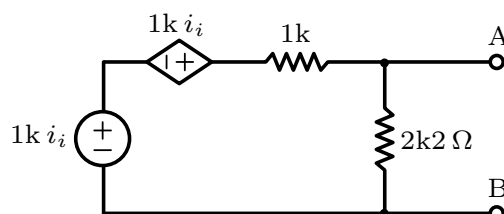


- Simplificação:
 - $1M01 \parallel 1k8 \simeq 1k8 \Omega$;
 - $8k2 \parallel 10k \simeq 4k5 \Omega$;
 - $v'_x/v_i = 1k8/1k9 \simeq 0.95 \text{ V/V}$;
 - $v_o/v'_x = -4k5/1k \simeq -4.5 \text{ V/V}$.

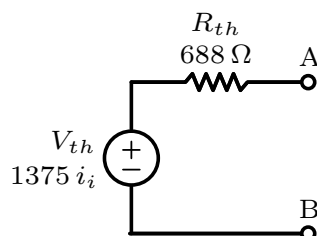
$$\therefore A_v = v_o/v_i = v_o/v'_x \cdot v'_x/v_i = -4.5 \cdot 0.95 \simeq -4.3 \text{ V/V}.$$

1.4

- a) • Norton \rightarrow Thévenin;

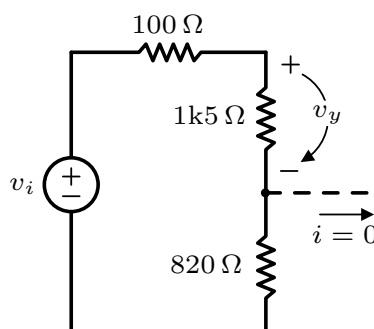


- $v_{AB} = (2k2/3k2) \cdot 2k i_i = 1375 i_i$;
- $R_{th} = 1k || 2k2 \simeq 688 \Omega$.



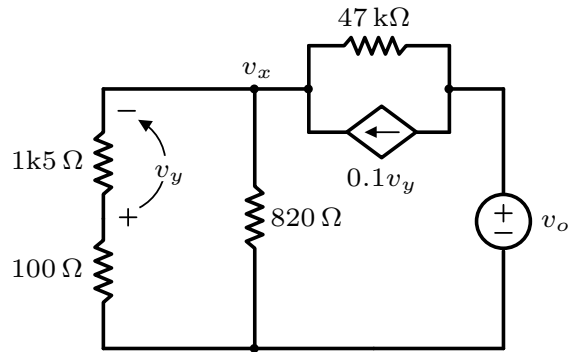
- b) • Em aberto, para cálculo de v_y basta olhar para apenas para a esquerda do circuito:

$$v_y/v_i = \frac{1k5}{100 + 1k5 + 820} \simeq 0.62 \text{ V/V}$$



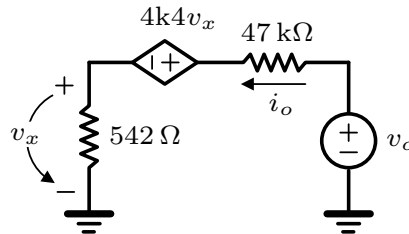
- Cálculo de V_{th} :

$$V_{th} = \frac{v_y}{1k5} \cdot 820 - 47k \cdot 0.1v_y \simeq -4k7 \cdot v_y \simeq 2k9 v_i$$



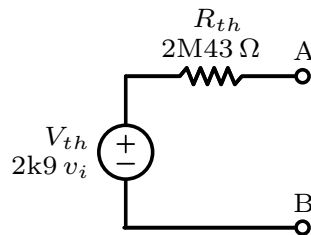
• Cálculo de R_{th} :

- $v_y = -(1k5/1k6)v_x$;
- $0.1v_y \cdot 47k = -4k4v_x$;
- $1k6 \parallel 820 \simeq 542\Omega$.



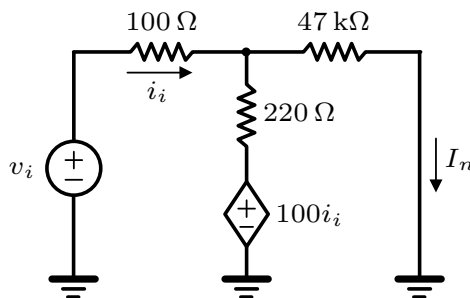
- $v_x = 542 i_o$; segue-se a aplicação do teorema da absorção da fonte:

$$R_{th} = 47k + 4k4 \cdot 542 + 542 \simeq 2M43\Omega$$

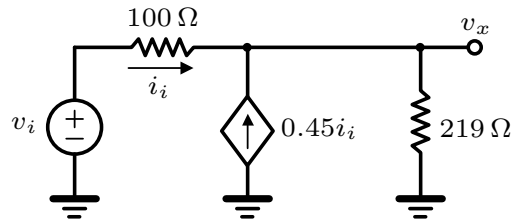


1.5

a) Cálculo da corrente de Norton, I_n



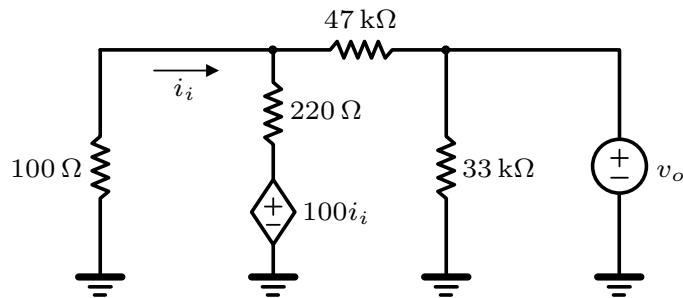
- Transformação auxiliar Thévenin \rightarrow Norton:
 - $V_{th} = (100/220) \cdot i_i = 0.45 i_i$;
 - $47k \parallel 220 \simeq 219 \Omega$.



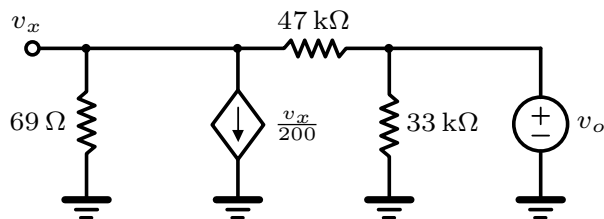
- Dual do teorema de Miller:
 - $\alpha = 0.45 \cdot i_i/i_i = 0.45$;
 - $R_{m1} = 219 \cdot (1 + \alpha) \simeq 318 \Omega$;
 - $v_x = (318/418) \cdot v_i \simeq 0.76 v_i$.

$$I_n = \frac{v_x}{47k} = \frac{0.76}{47k} v_i = 16.2 \mu \cdot v_i$$

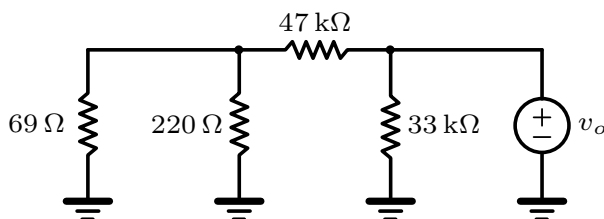
Cálculo da resistência de Norton, R_n



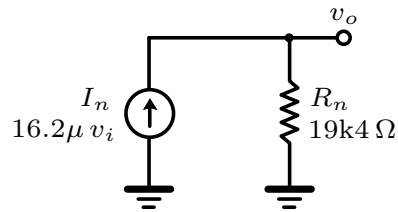
- $v_x = -100 i_i$;
- $200 \parallel 100 \simeq 69 \Omega$.



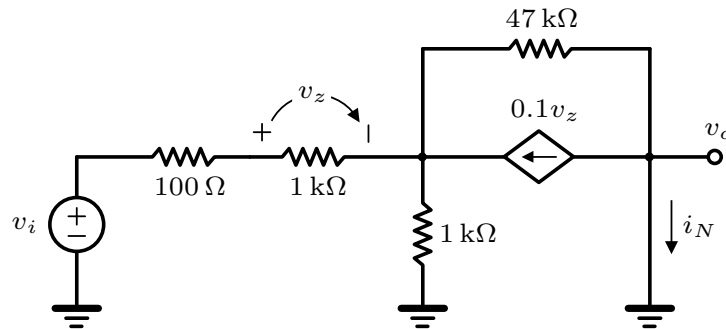
- Absorção da fonte $R_x = v_x/(v_x/200) = 200 \Omega$.



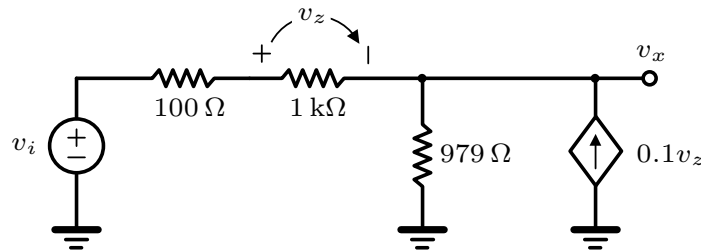
$$R_n = [(69 \parallel 220) + 47k] \parallel 33k \simeq 19k40 \Omega$$



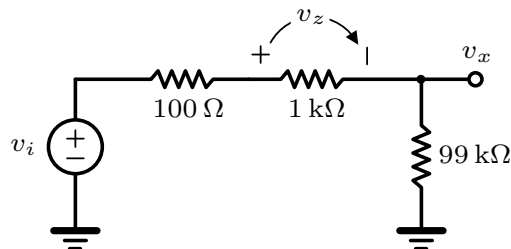
b) Cálculo da corrente de Norton, I_n



- $47k \parallel 1k \simeq 979 \Omega$;



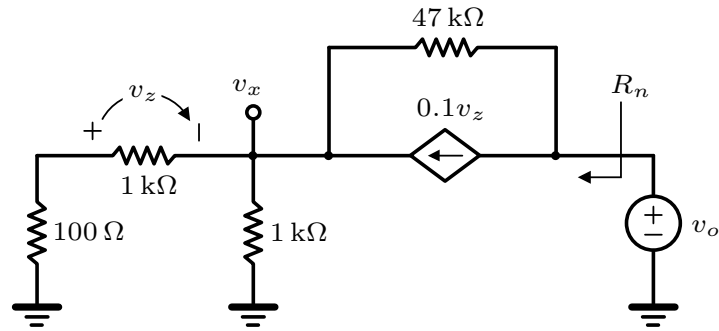
- Dual do teorema de Miller:
 - $\alpha = 0.1v_z/(v_z/1k) = 100 \text{ A/A}$;
 - $R_{m1} = 979 \cdot 101 \simeq 99 \text{ k}\Omega$.



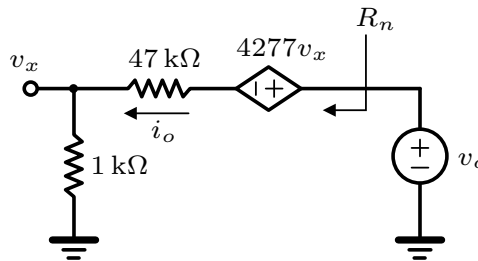
- $v_x/v_i = 99/100.1 = 998 \text{ mV/V}$;
- $v_z/v_i = 1/100.1 = 10 \text{ mV/V}$.

$$i_N = \frac{v_x}{47k} - 0.1v_z = \frac{989m}{47k} \cdot v_i - 0.1 \cdot 10m \cdot v_i \simeq -1m \cdot v_i$$

Cálculo de resistência de Norton, R_n

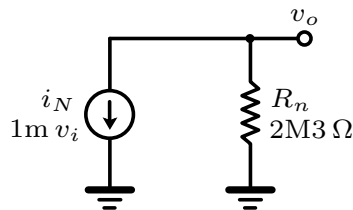


- $v_z = -1\text{k}/(100 + 1\text{k})v_x \simeq -0.91v_x$;
- $0.1v_z \cdot 47\text{k} = -0.91 \cdot 0.1 \cdot 47\text{k} \cdot v_x \simeq -4277v_x$;
- $1\text{k} || 1\text{k} \simeq 523 \Omega$.



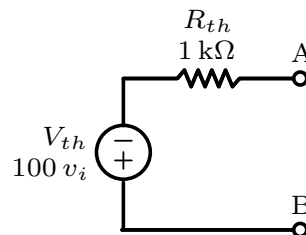
- $v_x = 523i_o$;
- $4277v_x = 4277 \cdot 523 \cdot i_o = 2\text{M}24 \cdot i_o$.

$$R_n = 523 + 47\text{k} + 2\text{M}24 \simeq 2\text{M}3 \Omega$$

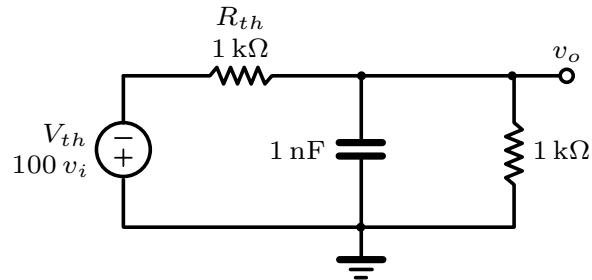


1.6

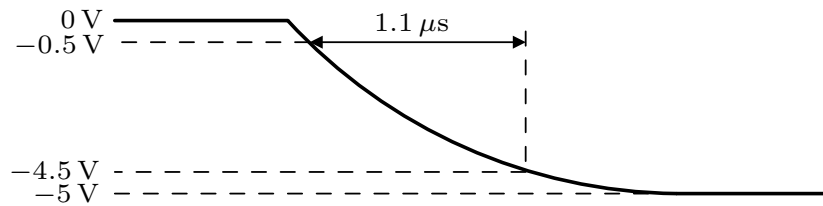
- a
- $v_o = -100 \cdot i_i \cdot 1\text{k} = -100 \cdot (v_i/1\text{k}) \cdot 1\text{k} \Leftrightarrow v_o/v_i = -100 \text{ V/V}$;
 - $v_i = 0 \Rightarrow i_i = 0 \Rightarrow R_{th} = 1 \text{ k}\Omega$.



b)



- $\tau = 1 \text{ nF} \cdot (1\text{k} \parallel 1\text{k}) = 500 \text{ ns}$;
- $f_c = 1/(2\pi\tau) \simeq 318 \text{ kHz}$;
- $t_r = 0.35/f_c \simeq 1.1 \mu\text{s}$.
- entrada de 0 a 100 mV $\Rightarrow V_{th}$ de 0 a $-100 \cdot 100 \text{ mV} = -10 \text{ V}$ e saída entre 0 e -5 V devido ao divisor de tensão.



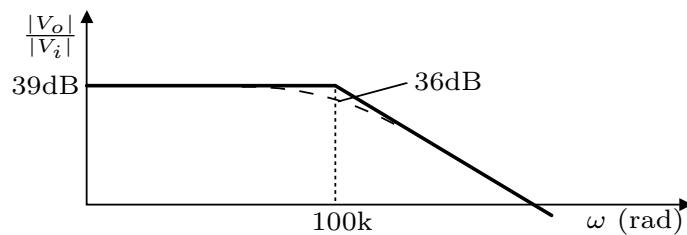
1.7

a) $R_i = 100 + (1\text{k} \parallel [-v_d/(-0.1v_d)]) = 100 + (1\text{k} \parallel 10) \simeq 110 \Omega$.

- b)
- $v_d \simeq -(10/110) \cdot v_i \simeq 90.9\text{m} \cdot v_i$;
 - $Z_o(s) = 10\text{k}/(1 + s10\text{k} \cdot 1\text{n})$;
 - $V_o = -0.1v_dZ_o = -90.9/(1 + s10\text{k} \cdot 1\text{n}) \cdot V_i(s)$

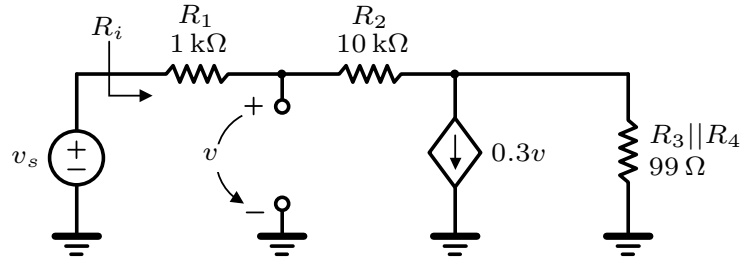
$$\frac{V_o(s)}{V_i(s)} = -\frac{90.9}{1 + s10\mu}$$

- polo: $-1/10\mu = -100 \text{ krad/s}$;
- ganho dc: $20 \log_{10}(90.9) \simeq 39 \text{ dB}$.

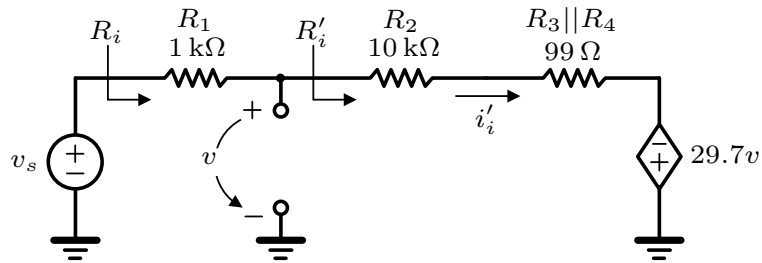


1.8 a) Cálculo de R_i

- $10k \parallel 100 \simeq 99 \Omega$;
- $30v/100 = 0.3v$.

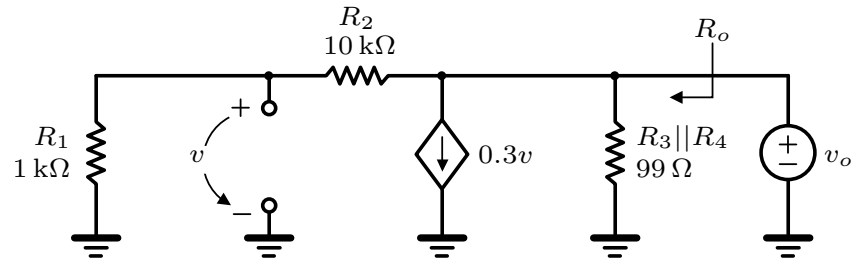


- $0.3v \cdot 99 = 29.7v$.

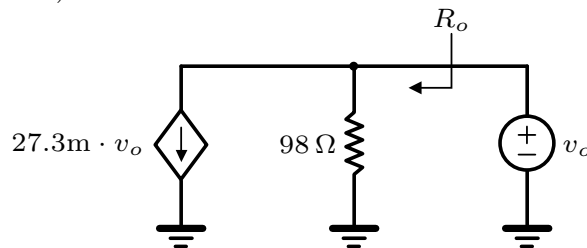


$$\therefore R'_i = \frac{v}{i'_i} = \frac{v}{(v+29.7v)/(10k+99)} \simeq 329 \Omega \Rightarrow R_i = 1k + R'_i = 1329 \Omega.$$

Cálculo de R_o

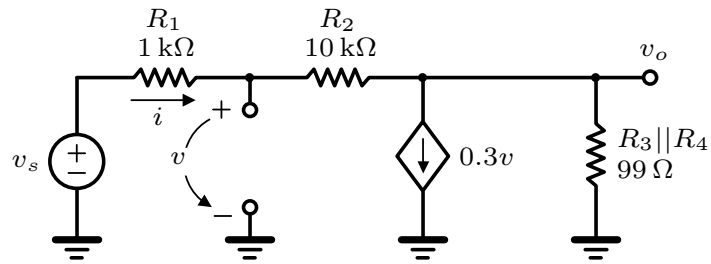


- $v = \frac{1k}{1k+10k}v_o = v_o/11$;
- $0.3v = 0.3v_o/11 \simeq 27.3m \cdot v_o$;
- $99 \parallel (10k + 1k) \simeq 98 \Omega$.



$$\therefore R_o = \frac{1}{27.3m} \parallel 98 \simeq 27 \Omega.$$

b



- $v = \frac{R_i - 1k}{R_i} v_s = \frac{329}{1329} v_s$;
 - $i = \frac{v_s}{R_i} = \frac{v_s}{1329}$;
 - $v_o = (i - 0.3v) \cdot 99 = \left(\frac{1}{1329} v_s - 0.3 \cdot \frac{329}{1329} v_s\right) \cdot 99$.
- $\therefore A_v = v_o/v_s \simeq -7.3 \text{ V/V}$.

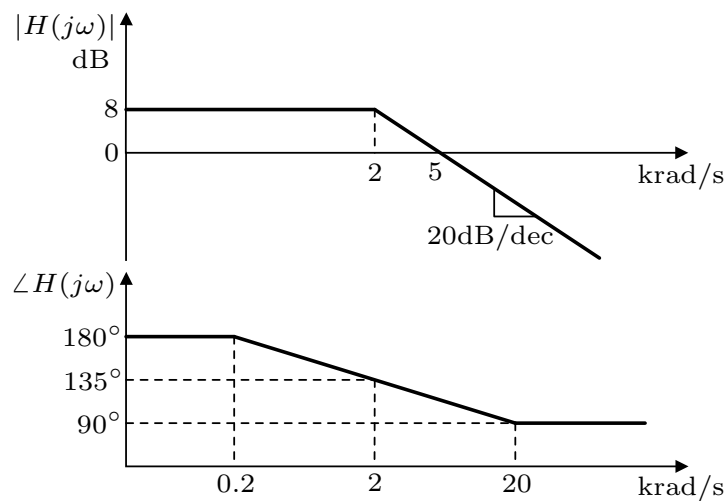
c

- $Z_L = \frac{R_6}{1+sR_6C} = \frac{100}{1+s \text{ 1m}}$.

$$\frac{V_{o1}}{-5V_s} = \frac{Z_L}{R_{th} + R_5 + Z_L} = \frac{\frac{100}{1+s \text{ 1m}}}{100 + \frac{100}{1+s \text{ 1m}}}$$

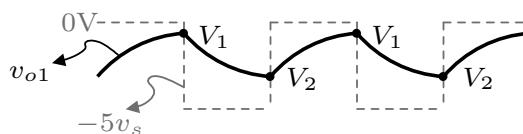
$$H(j\omega) = \frac{V_{o1}(j\omega)}{V_s(j\omega)} = -\frac{5}{2 + j\omega \text{ 1m}} = -\frac{2.5}{1 + j\frac{\omega}{2k}}$$

- $|H(0)| = 20 \log_{10}(2.5) \simeq 8 \text{ dB}$;
- polo: $\omega_p = 2/1\text{m} = 2 \text{ krad/s}$ (318 Hz).



$$\boxed{d} \quad \tau = [R_6 \parallel (R_{th} + R_5)] \cdot C = 500 \mu s = T/4.$$

Portanto, $T/2 \ll 5\tau \Rightarrow C$ não carrega totalmente.



- Os valores finais em cada fase são $0V$ e $-5/2 = -2.5V$ devido ao divisor resistivo $R_{th} + R_5 = R_6$;
- $V_1 = 0 - (0 - V_2) \cdot e^{-(T/2)/\tau} = V_2 \cdot e^{-(T/2)/\tau} = V_2 e^{-2}$;
- $V_2 = -2.5 - (-2.5 - V_1) \cdot e^{-(T/2)/\tau} = -2.5 + (2.5 + V_1) \cdot e^{-2}$.

$$V_2 = -2.5 \cdot \frac{1 - e^{-2}}{1 - e^{-4}} = -2.2V \quad \Rightarrow \quad V_1 = -0.3V$$

4. Soluções Numéricas dos Exercícios Propostos

2.1 $R_i = 0 \Omega$.

2.2 $R_i = -933 \text{ k}\Omega$.

2.3 —

2.4 (a) $A_v = 112 \text{ V/V}$; (b) $R_{\text{sig}} \simeq 85 \Omega$.

2.5 $A_{vo} = -448 \text{ V/V}$.


2.6 (a) 42 V/V ; (b) -19.2 V/V ; (c) -22.8 V/V .


















































2.7 (a) $V_{th} = 7.9i_i$, $I_n = \infty$, $R_{th} = R_n = 0 \Omega$;
(b) $V_{th} = -9.65v_i$, $I_n = 0.152v_i$, $R_{th} = R_n = 63.4 \Omega$.

2.8 (a) $R_i = 1 \text{ k}\Omega$, $R_o = 40 \text{ k}\Omega$, $A_v = 2000 \text{ V/V}$;
(b) $A_v(0) = 60 \text{ dB}$, $w_p = 50 \text{ krad/s}$.

2.9 (a) $R_{th} = 98 \Omega$, (b) $\tau = 739 \mu\text{s}$, $t_s = 1.625 \text{ ms}$.

5. Ficheiros de Simulação

Para aceder aos ficheiros de simulação de cada exercício, clique no símbolo  associado ao respetivo exercício. Todos os circuitos foram implementados na versão 12 do MULTISIM para MS Windows e em Synopsys HSPICE J-2014.09-SP1-1 64-BIT, Linux.

	Multisim	Hspice		Multisim	Hspice
1.1 a			2.1		–
1.1 b			2.2		–
1.1 c			2.3		–
1.1 d			2.4 a		–
1.2 a			2.4 b		–
1.2 b			2.5		–
1.3 a			2.6 a		–
1.3 b			2.6 b		–
1.3 c			2.6 c		–
1.4 a			2.7 a		–
1.4 b			2.7 b		–
1.5 a			2.8 a		–
1.5 b			2.8 b		–
1.6 a		–	2.9 a		–
1.6 b		–	2.9 b		–
1.7 a		–			
1.7 b		–			
1.8 a		–			
1.8 b		–			
1.8 c		–			
1.8 d		–			